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Abstract

Operational modal analysis is an important step in many methods for vibration-based structural health monitoring. These methods provide the modal parameters (frequencies, damping ratios and mode shapes) of the structure and can be used for monitoring over time. For a continuous monitoring the excitation of a structure is usually ambient, thus unknown and assumed to be noise. Hence, all estimates from the vibration measurements are realizations of random variables with inherent uncertainty due to unknown excitation, measurement noise and finite data length. Estimating the standard deviation of the modal parameters on the same dataset offers significant information on the accuracy and reliability of the modal parameter estimates. However, computational and memory usage of such algorithms are heavy even on standard PC systems in Matlab, where reasonable computational power is provided. In this paper, we examine an implementation of the covariance-driven stochastic subspace identification on the wireless sensor platform PEGASE, where computational power and memory are limited. Special care is taken for computational efficiency and low memory usage for an on-board implementation, where all numerical operations are optimized. The approach is validated from an engineering point of view in all its steps, using simulations and field data from a highway road sign structure.

1 INTRODUCTION

The design and maintenance of mechanical or civil structures subject to noise and vibrations is an important topic in structural engineering. Laboratory or in-operation vibration tests are performed on structures for modal analysis, where modal models are identified containing the vibration modes (frequencies, damping ratios, mode shapes) related to the poles and observed eigenvectors of a linear time-invariant system. Stochastic subspace identification (SSI) methods have been proven efficient for their identification from output-only measurements for operational modal analysis (OMA) [1,2]. In particular, these methods offer excellent theoretical and practical properties and can be used in wide-spread application areas. Amongst others, automated [3] and computationally efficient implementations [4] are available. All modal parameter estimates from the vibration measurements are afflicted with uncertainty due to unknown ambient excitation, measurement noise and finite data length.

The corresponding variance of the modal parameters is an important information to assess their accuracy and to allow a meaningful comparison of modal parameters during monitoring. It can be estimated with computationally efficient methods [5,6].

On the technological side, the field of smart wireless sensor systems performing real-time monitoring is developing quickly. With the practical difficulties to access structures, to collect data and then perform off-line and remote computation, embedded wireless sensor networks (WSN) offer an advantage compared to classical measurement systems and are an important contribution towards self-monitored structures. In particular, the PEGASE platform [7] offers high level functions to perform wireless data collecting and time stamping up to few microseconds UTC.

The practical implementation of SSI based modal analysis together with the computation of the uncertainty bounds of the identified modal parameters on the PEGASE platform is investigated in this paper. The computation of uncertainty bounds is an advantage compared to comparable implementations [8,9]. A challenge is the limited computational power and memory on the sensor platform, while even on a PC the computational requirements are heavy in particular for uncertainty quantification. Special care is taken for computational efficiency and low memory usage for an on-board implementation, where all numerical operations are optimized. In particular, the uncertainty quantification has a high computational complexity and memory demands, whose optimization has been a challenge [5,6]. The requirements are lowered from a PC Matlab implementation to a low memory constrained embedded system for industrial applications. The approach is validated from an engineering point of view in all its steps, using simulations and field data from a highway road sign structure.

2 SUBSPACE-BASED SYSTEM IDENTIFICATION AND VARIANCE COMPUTATION

2.1 Models and parameters

The vibration behavior of a mechanical system is assumed to be described by a stationary dynamical system

$$\begin{aligned} \mathbf{M}\ddot{z}(t)+\mathbf{C}\dot{z}(t)+\mathbf{K}z(t)&=v(t), \\ y(t)&=L\ddot{z}(t)+w(t), \end{aligned} \tag{1}$$

where t denotes continuous time, $\mathbf{M},\mathbf{C},\mathbf{K}\in\mathbb{R}^{m\times m}$ are the mass, damping and stiffness matrices, vector $z\in\mathbb{R}^{m}$ collects the displacements of the degrees of freedom of the structure, the non-measured external force v modeled as white noise, the measurements at the sensors are collected in the vector $y\in\mathbb{R}^{r}$, matrix L indicates the sensor locations, and w denotes white measurement noise. The number of degrees of freedom are denoted by m, and r is the number of sensors. Note that in (1) the measurements are accelerations, but velocities or displacements are also possible.

Sampling model (1) at time steps $t=k\tau$ yields a discrete-time state space model

$$\begin{aligned} x_{k+1}&=Ax_k+v_k \\ y_k&=Cx_k+w_k \end{aligned} \tag{2}$$

with state transition matrix $A \in \mathbb{R}^{n \times n}$ and output matrix $C \in \mathbb{R}^{r \times n}$ with model order $n = 2m$. The modal parameters of system (1) are equivalently found from the eigenstructure of (2), $A\phi_i = \lambda_i \phi_i$, as

$$\mu_i = \frac{\log(\lambda_i)}{\tau}, \quad f_i = \frac{|\mu_i|}{2\pi}, \quad \xi_i = \frac{-\mathrm{Re}(\mu_i)}{|\mu_i|}, \quad \varphi_i = C\phi_i \tag{3}$$

where f_i is the natural frequency, ξ_i the damping ratio and φ_i the mode shape of mode *i*.

2.2 Stochastic subspace identification

The goal of subspace identification is to estimate the matrices *A* and *C* from the output measurements y_k, $k = 1, \ldots, N$, in system (2). Then, the modal parameters are obtained from (3). In this work, we use the covariance-driven subspace identification, consisting of the following steps.

From the measurements, the output covariance estimates R_i are computed for $i = 1, \ldots, 2p$, where *p* is chosen such that $pr \geq n$, and filled into a Hankel matrix

$$\mathcal{H} = \begin{pmatrix} R_1 & R_2 & \cdots & R_p \\ R_2 & R_3 & \cdots & R_{p+1} \\ \vdots & \vdots & \ddots & \vdots \\ R_{p+1} & R_{p+2} & \cdots & R_{2p} \end{pmatrix}, \quad R_i = \frac{1}{N-i} \sum_{k=1}^{N-i} y_{k+i} y_k^T .$$

The matrix $\mathcal{H}$ possesses the factorization property $\mathcal{H} = \mathcal{O}\mathcal{C}$ into observability and controllability matrix, where

$$\mathcal{O} = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^p \end{pmatrix}.$$

This matrix is obtained from a singular value decomposition (SVD) of $\mathcal{H}$ from the *n* left singular vectors. Then, matrix *C* is retrieved from the first block row of $\mathcal{O}$, and *A* is obtained from a least squares solution of

$$\mathcal{O}^{\uparrow} A = \mathcal{O}^{\downarrow} \tag{4}$$

where $\mathcal{O}^{\uparrow}$ and $\mathcal{O}^{\downarrow}$ are obtained from $\mathcal{O}$ by removing the last and first block row, respectively. Finally, the modal parameters are retrieved from (3).

2.3 Variance computation

The modal parameter estimates are afflicted with statistical uncertainty due to unknown ambient excitation, measurement noise and finite data length. Computing the variance of the estimates is essential, e.g. to assess their accuracy or to compare estimates from different datasets during monitoring. In [5,6,10] a method for the computation of the model parameter variance from covariance-driven subspace identification is detailed, where both the modal parameters and their variance are obtained from the same dataset.

In this approach, the sample covariance of the Hankel matrix is propagated to the desired parameters by considering a sensitivity analysis. The required sensitivities are derived analytically through the propagation of a first-order perturbation from the data to the identified parameters. For any vector-valued function of the Hankel matrix, in particular for the modal parameters, it holds

$$\Delta f(\mathcal{H}) = \mathcal{J}_{f,\mathcal{H}} \Delta \mathrm{vec}(\mathcal{H}) \;\Rightarrow\; \mathrm{cov}(\mathrm{vec}(\mathcal{H})) \approx \mathcal{J}_{f,\mathcal{H}} \mathrm{cov}(\mathrm{vec}(\mathcal{H})) \mathcal{J}_{f,\mathcal{H}}^T \tag{5}$$

where $\mathcal{J}_{f,\mathcal{H}}$ denotes the sensitivity of f with respect to $\mathrm{vec}(\mathcal{H})$, and vec is the column stacking vectorization operator.

This approach is computationally feasible, since the sample covariance $\mathrm{cov}(\mathrm{vec}(\mathcal{H}))$ is easily computed by cutting the dataset into blocks, and the sensitivities are computed using the estimates from system identification. However, the size of the involved covariance matrices can get huge in practice, since it is squared in comparison to the size of the underlying variable. This makes a direct implementation of the covariance computation in (5) impossible for realistic problem sizes both in terms of memory usage and computational complexity. An efficient implementation is given in detail in [5,6], which allows the computation for model orders in the 100’s in less than a few minutes on a PC.

2.4 Efficient computation for multiple model orders

In operational modal analysis (OMA), the model order of the system is in general unknown and needs to be overestimated due to noise. The stabilization diagram is a standard tool for OMA, plotting the system identification results for multiple model orders. This allows to separate the true structural modes from spurious noise modes, since the latter tend to vary for different orders.

However, the computation of the modal parameters and their variances for an entire stabilization diagram is a computationally demanding task, since the least squares problem for the system matrix in (4) has to be solved for every system order. The computational complexity can be reduced significantly by exploiting the structure of the observability matrix for different model orders in the solution of (4), which is detailed for system identification in [4] and for the variance computation in [5,6]. Then, the entire stabilization diagram can be computed with the same computational complexity as only one system identification at the maximal model order.

With these optimizations, an efficient implementation of the modal analysis algorithm and the variance computation becomes possible. For an implementation in environments with restricted memory, such as the embedded wireless sensor platform PEGASE in the following, special care is taken for optimal memory usage.

3 WIRELESS SENSOR PLATFORM PEGASE FOR IMPLEMENTATION

PEGASE is the commercial name of a generic Wireless Sensor Platform conceived and designed by IFSTTAR since 2008. The PEGASE concept is essentially based on a generic vision of its hardware and software abilities. Hardware genericity is provided by a principle of mother and pluggable daughter boards. The PEGASE mother board (described below) integrates most common functions of typical wireless systems: ensure computation, manage energy, offer multiple I/Os and wireless communications. Each pluggable daughter board adds a specific function to the mother board, such as 8-analog/digital channels, 3/4G GSM extension, or inertial measurements. Software genericity is embedded through a small Linux Operating System added to an open Single Development Kit (SDK) given in open-source in object-oriented languages.

A first generation of PEGASE 1 has been designed and sold by a third-party company of IFSTTAR in thousands of units since 2008. It is used in many SHM applications [7], such as acoustic monitoring of bridge cables, strain gauges monitoring, vibration monitoring, etc.

As electronics is a domain subject of fast evolutions, a new generation of PEGASE is about to be industrialized in 2016. PEGASE 2 (Fig. 1 and 2) is not only a more efficient electronic device, but it is also linked to a *cloud* supervision software that allows to operate various sensors (PEGASE and not only).

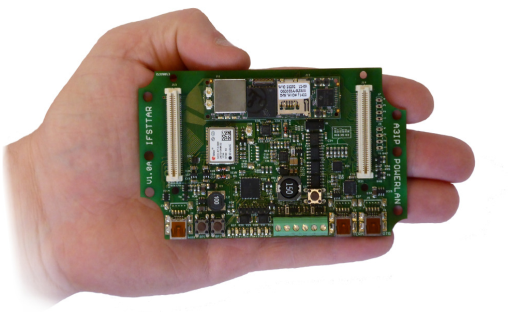


Figure 1: Generic wireless sensor platform PEGASE 2

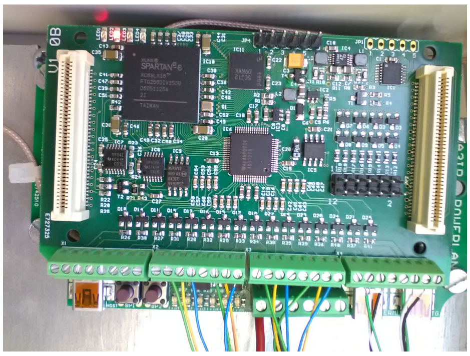


Figure 2: 8 channel analog daughter board to collect "real" data

The major evolved characteristics of PEGASE 2 are:

- Extended generic behavior: by means of a Linux (Debian) embedded operating system and a free Single Development Kit (SDK) in C++ language. Each physical processor interruption can be linked to user C++ methods
- Accurate absolute time-synchronization up to 100 nanoseconds UT based on a GPS/PPS receiver and real-time algorithm in a driver
- Native power-manager for Lithium batteries and solar cells inputs
- Native GPS receiver NEO-6T from Ublox for geolocalization and accurate time synchronization
- Native 3D MEMS for inertial measurements
- Integration of Gumtix Overo® FE COM as core module for: processing (DSP TI5330 from Texas Instrument), WiFi/Bluetooth LTE wireless communications, 32GB by SD memory storage and 16MB of RAM...
- A high capacities 8 analog (accelerometers, strain gauges, temperature, ...) and 8 digital inputs daughter board. Differential analog inputs range +/-5 V; 24 bits of digitalization; up to 144 kHz of sampling frequency
- A set of other plug-and-play daughter boards: Ethernet, 3/4G GSM communications, inertial measurements...

The previous implementation of SSI algorithms on PEGASE 1 gave promising but limited results [8] due to the memory limitations. PEGASE 2 offers increased perspectives for embedded computation of SSI including uncertainty quantification in a more efficient environment.

4 VALIDATION AND APPLICATION OF EMBEDDED SSI ON PEGASE 2

Compared to the previous implementation on PEGASE 1 [8], it was not necessary to implement the fast SSI methods from scratch in C language using Lapacke libraries on PEGASE 2. Instead, available Matlab code of the efficient SSI and uncertainty quantification was implemented in C using the Matlab Coder toolbox, and the code was carefully adjusted “by hand” in order to adapt it correctly to the PEGASE 2 platform. The applicability of the code to various structures and user-defined environments is ensured by an external set of parameters. Furthermore, a development task has been carried out to operate a flow of real data coming from the analog inputs of the 8 analog input daughter board.

The validation of the embedded implementation on the PEGASE 2 platform has been carried out in three steps:

1. Comparison between identical datasets run in Matlab and on PEGASE 2
2. Validation on a pre-defined artificial vibration signal fed to PEGASE 2
3. Application on a real road-sign structure under ambient vibration monitoring

The results of this validation campaign are described below.

4.1 Validation Matlab/PC versus C/PEGASE2

The first validation task consisted in running the algorithms using the same benchmark of data on the two platforms (Matlab on PC versus C on PEGASE 2). The correlation between the two platform implementations is perfect.

Process time on PEGASE 2 is about 165 s for a benchmark consisting of a matrix of 15 row (e.g. 15 sensors) by 100 000 lines. This corresponds, at 100 Hz of sampling frequency, to 1000 seconds of data acquisition.

4.2 Validation using a generated analog signal

In order to validate the whole acquisition and modal parameter computation chain on PEGASE 2, an emulation platform has been set up. A known damped signal was generated electrically on the analog inputs of the PEGASE 2 system. The analog signal was also digitalized and given through files to Matlab fast SSI algorithm for a cross validation. This free vibration signal was applied on the 8 analog inputs with following properties:

$$a(t) = a_0 \exp(-\omega\xi t)\cos(\sqrt{1-\xi^2}\,\omega t) \tag{4}$$

where a_0 is the maximal amplitude, $\omega = 2\pi f$ with $f = 10$ Hz is the frequency and $\xi = 0.02$ is the damping ratio. Fig. 3 shows the stabilization diagram obtained from the analog signal, where the frequency is correctly estimated.

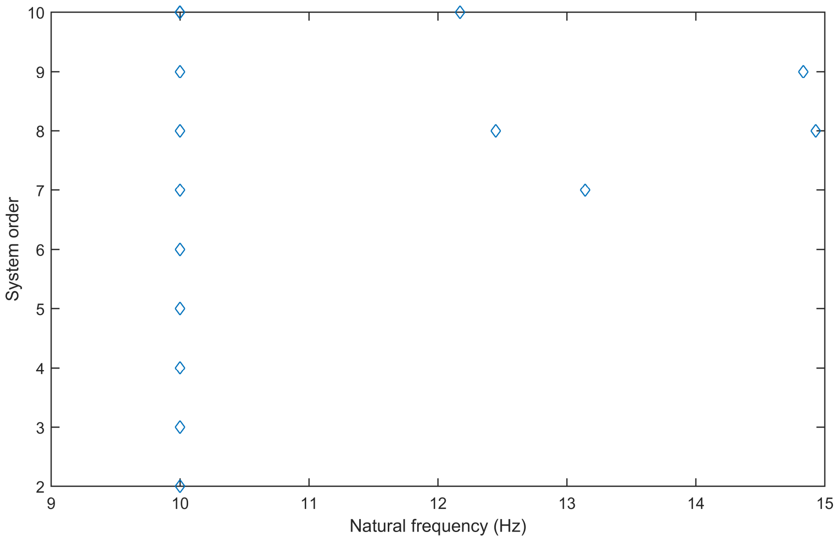


Figure 3: Stability diagram for a real analog damped signal.

4.3 Application on a real road-sign structure

Structural health monitoring of road sign structures is indeed a relevant subject for road managers due to several recent accidents [11]. At the IFSTTAR laboratory located in Nantes, a real road-sign structure is available for research and development activities (Fig. 4). Furthermore, a finite element model of the structure has been established (Fig. 5).

Figure 4: Road sign structure at IFSTTAR

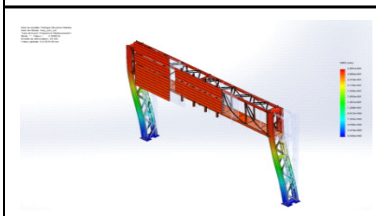
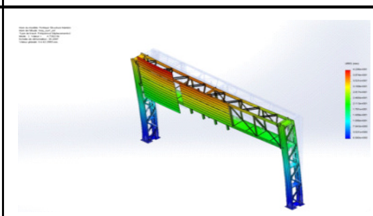
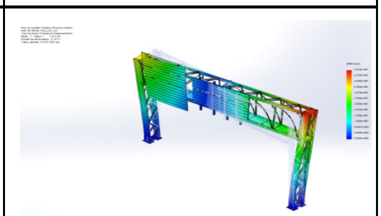
Mode 1	Mode 2	Mode 3
		
4.36 Hz	4.56 Hz	7.83 Hz

Figure 5: Model of road sign structure

For the field application to the road sign structure, a PEGASE 2 platform and six mono-axis accelerometers (Silicon Design M2210) have been installed on the structure (Fig. 6). Since April 2016, the system is running, sending periodically modal parameters from SSI directly to a web could platform via wireless IP (WiFi + 3G) connection. Modal parameters are identified, every 30 minutes, from data packages of 300 seconds at a sampling frequency of 100 Hz (30,000 data samples). The fast SSI algorithm runs in less than 25 seconds. This allows the system to run, if needed, continuously because PEGASE processes data faster than the acquisition period. The computation time is shorter than in the benchmark computation in Section 4.1, since less sensors (6 instead of 15) and less data (30,000 instead of 100,000 samples) are used.

Most of parameters for the algorithm can be customized from the webserver (Fig. 7): sampling frequency, number of channels (1 to 8), reference channels, min and max order, ...

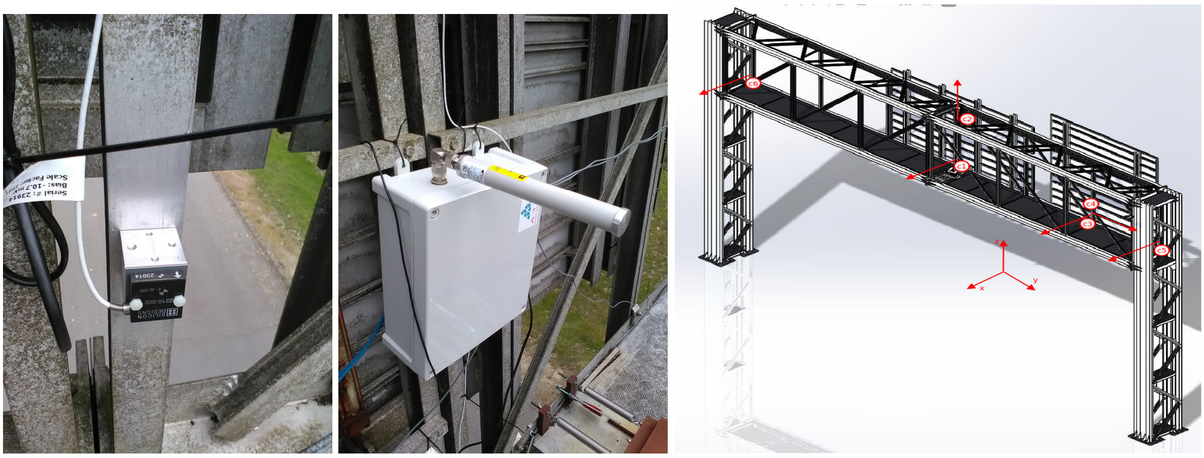


Figure 6: Installation of PEGASE 2 and sensors on IFSTTAR gantry.

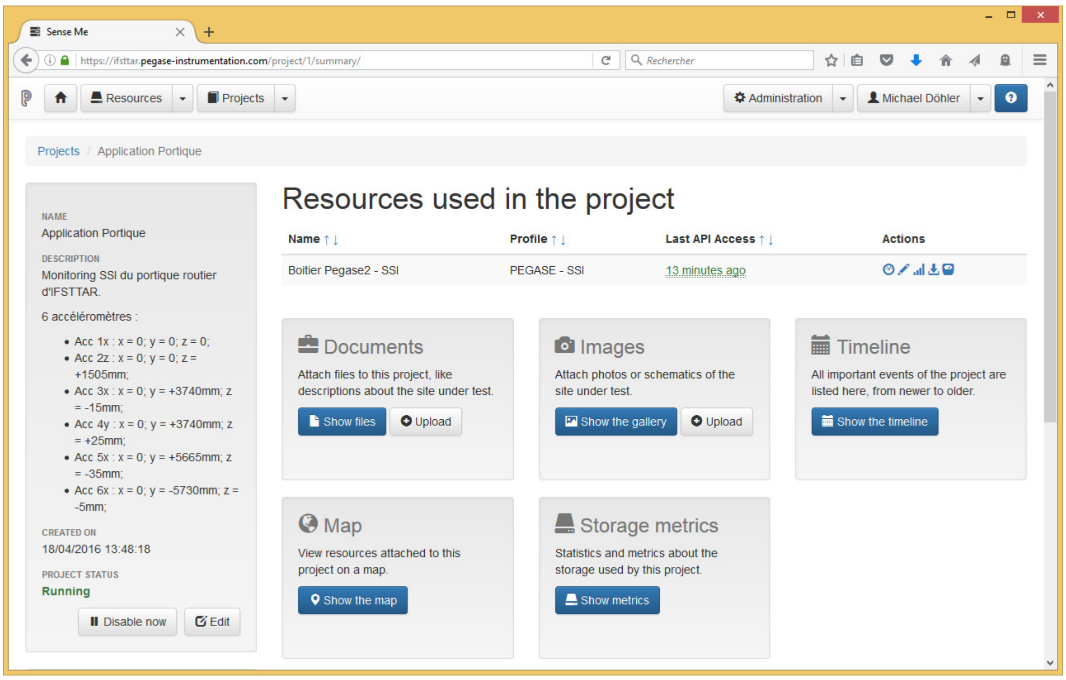


Figure 7: Screenshot of server web interface.

The resulting stabilization diagram from the data processing of one measured dataset on the road sign structure, containing also the uncertainty bounds, are shown in Fig. 8. The modal parameters of the first three identified modes are detailed in Table 1, where their coefficient of variation are shown (the estimated standard deviations, divided by the nominal value of the respective parameter). Damping ratios have much larger uncertainties than frequencies (in relative terms), which is according to statistical theory [12]. The first three modes correspond quite well to the numerical model of the road sign structure as shown in Fig. 6.

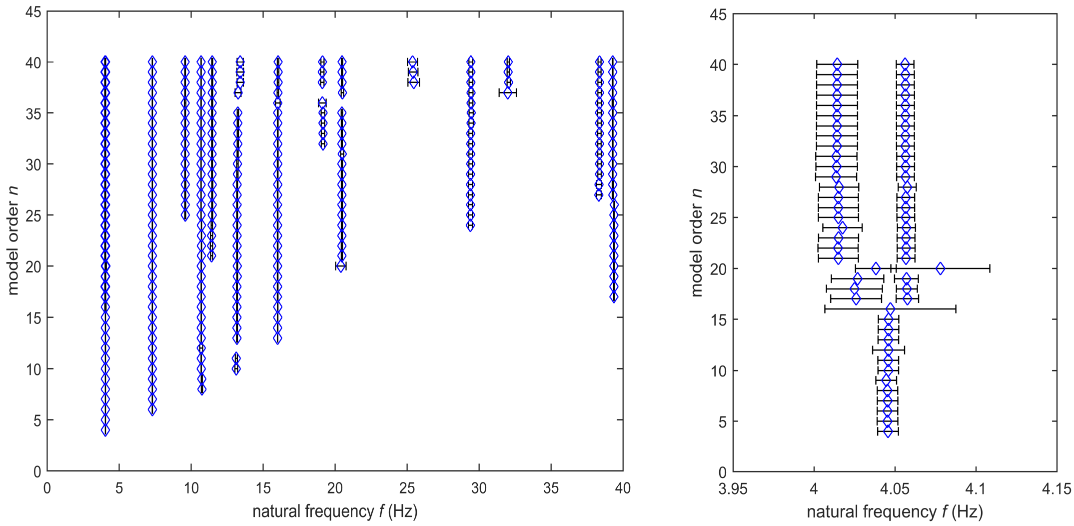


Figure 8: Stabilization diagram with ±1σ uncertainty bounds (left), zoom on first two modes (right).

Table 1: Identified modal parameters of first three modes and their estimated coefficient of variation.

mode	f (Hz)	f / σ_f (in %)	ξ (%)	ξ / σ_ξ (in %)
1	4.01	0.32	0.7	34
2	4.06	0.13	0.7	20
3	7.30	0.13	0.9	22

5 CONCLUSIONS

In this article, the results of the implementation of the fast SSI algorithm on the efficient and smart industrial wireless platform PEGASE 2 were presented. The Matlab algorithms were ported to C for the wireless platform and the implementation was validated carefully in each step, comparing between Matlab and PEGASE results in a benchmark, an emulated analog signal and on a real road sign structure under ambient conditions. Further validation will be carried out in the future, also regarding the simulation of structural defaults by adding a mass to the road sign structure. A specific step, will consist in the computation of the uncertainty bounds of the identified modal parameters.

Transcoding Matlab code to C language using the Matlab Coder toolbox turned out to be an important gain in terms of implementation up to some corrections 'by hand'. The system maturity still seems high enough and represents a first step towards transfer to industry. A more efficient direct implementation in C is nevertheless required for full optimization and will follow.

The wireless implementation in a small and low-cost box in combination with a cloud web server contributes to the *Internet of Things* approach of structural health monitoring.

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